

IPPP/03/47  
DCPT/03/94  
August 2003

# QCD Sum Rules on the Light-Cone, Factorisation and SCET

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## Abstract:

The accurate calculation of weak heavy-to-light form factors is crucial for the determination of CKM matrix elements from experimental data on B decays. In SCET, the soft-collinear effective theory, these form factors can, in the heavy quark limit, be split into a factorisable part that is calculable in perturbation theory, and a nonfactorisable part that observes certain spin-symmetries. I discuss the relation between the SCET factorisation formulas and the heavy quark limit of the corresponding QCD sum rules on the light-cone. I also analyse the numerical size of factorisable and nonfactorisable parts and corrections suppressed by powers of the b quark mass.

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# 1 Introduction and a Bit of History

The recent interest in B decay form factors into light mesons is driven, on the one hand, by the ever increasing accuracy of experimental results on semileptonic and rare decays obtained at BaBar and Belle, which calls for a match in theory, and on the other hand by recent theoretical developments that aim at a rigorous derivation of factorisation formulae for form factors in terms of perturbatively calculable scattering and kernels and meson distribution amplitudes on the light-cone. Most presently available numerical predictions for form factors come from either lattice calculations [1] or QCD sum rules on the light-cone [2]. The key characteristics of both methods is probably the fact that they give predictions for form factors at physical meson masses without any explicit reference to heavy quark expansion. A second, rather different approach is based on the interpretation of  $m_b$  as a large scale which warrants an expansion in  $1/m_b$  and makes it possible to draw on the power of factorisation theorems and (spin-) symmetry relations between form factors for different processes. The first attempt in this direction is probably Ref. [3], where  $B \rightarrow \pi$  was treated in a (as it turned out) naïve generalisation of the Brodsky-Lepage formalism for describing exclusive hard perturbative QCD processes [4]. The factorisation formula derived in [3] suffers from a divergence in the convolution of the perturbatively calculated amplitude with the leading nonperturbative contributions of the B and  $\pi$  mesons; divergences of this type have been termed *soft* or *endpoint* singularities. It was later argued in Ref. [5] that for decays of heavy mesons the perturbative hard-gluon exchange process whose dominance is assumed in the Brodsky-Lepage formalism is actually of the same order in a  $1/m_b$  expansion as the so-called soft Feynman mechanism, where not all partons in a meson participate in the hard subprocess – in particular configurations where the spectator quark stays soft, but the light quark produced in the weak decay has large energy are not suppressed by powers of  $1/m_b$  with respect to the hard-gluon exchange configuration, cf. Fig. 1. It was argued, though, that, as a highly asymmetric edge-of-phase-space configuration, soft contributions may be suppressed by Sudakov logarithms, but it was also argued that the logarithmic suppression may not be effective at the moderate scale  $m_b \sim 5$  GeV.

The results of Ref. [5] had no immediate impact and later papers, e.g. Ref. [6], continued to assume dominance of the hard mechanism and concentrated on the treatment of large logarithms, neglecting power-suppressed effects. The numerical results for form factors were very small and at variance with those from quark models, Ref. [7], and lattice

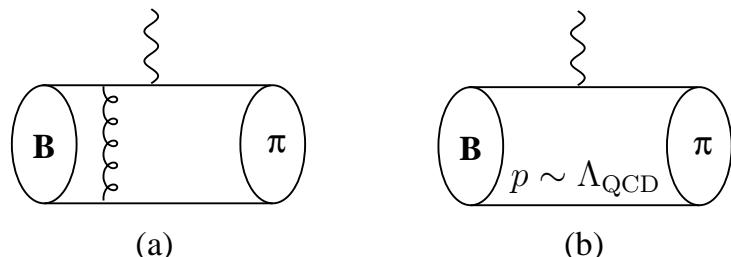


Figure 1: Hard-scattering contribution to  $B \rightarrow \pi$  form factors (a) vs. soft contribution (b).

calculations and also hardly compatible with experimental data on semileptonic B decays. At this point QCD sum rules on the light-cone (LCSRs) entered the game, a generalisation of the original QCD sum rule method of Shifman, Vainshtain and Zakharov [8] to processes with large momentum transfer. LCSRs are particularly apt to describe *both* soft *and* hard contributions to B decays into light mesons [2] – they can be characterised as combining (field-theoretical) rigour with a justifiable degree of model-dependence, which makes it possible to obtain numerical predictions at the physical (b quark) mass scale without recourse to heavy quark expansion, but at the same time also limits the accessible accuracy upon inclusion of higher corrections. The next step forward was stimulated by the success of heavy quark effective theory and its spin-symmetry relations for heavy-to-heavy form factors. Similar relations were found to apply to heavy-to-light decays, in the combined limits  $m_b \rightarrow \infty$  and  $E_\pi \rightarrow \infty$  and were formulated in the framework of LEET (large energy effective theory) [9], an effective field theory based on the combination of both heavy quark and large energy limit for light quarks. The LEET symmetry relations were in excellent numerical agreement with the results obtained from QCD sum rules on the light-cone, but neglected the (symmetry-breaking) effects from hard gluon exchange. The latter were first investigated on a diagrammatical level in Ref. [10] and later on formulated in the language of effective field theories in Ref. [11]; the resulting field theory has become known as SCET – soft-collinear effective theory. SCET is basically a further development of LEET that cures its major deficiency, its failure to reproduce the infrared structure of QCD, which makes it for instance impossible to properly match LEET onto QCD and results in the (re-)appearance of endpoint singularities in the hard-gluon exchange contributions to form factors. SCET is actually not one, but two effective theories with parametrically different cutoff scales  $O(m_b)$  and  $O(\sqrt{m_b \Lambda_{\text{QCD}}})$ , respectively, and different degrees of freedom: (ultra-)soft and collinear quark and gluon fields in  $\text{SCET}_I$  and (ultra-)soft fields in  $\text{SCET}_{II}$ . SCET provides a powerful framework to derive and study factorisation theorems in QCD, allowing one to resum large single and double logarithms in the respective cutoff scales: replacing the one-step matching  $\text{QCD} \leftrightarrow \text{LEET}$  by the two-step matching  $\text{QCD} \leftrightarrow \text{SCET}_I \leftrightarrow \text{SCET}_{II}$  removes endpoint singularities (at least to leading order in the expansion) and allows one to use RG-techniques to resum (both single and double) logarithms in the parametrically large ratio of cutoff scales. On the other hand, like with all effective field theories, a systematic expansion in inverse powers of the cutoff is possible, but becomes rather involved, cf. [12, 13]. As far as phenomenology is concerned, the question if the correct treatment of logarithmic terms  $\sim \ln(m_b/\sqrt{m_b \Lambda_{\text{QCD}}})$  at the price of a considerable complication in the treatment of power-suppressed terms is justified is a legitimate one – and yet unanswered. It is indeed this question which was one of the main motivations for writing this paper.

One of the major achievements of SCET is the elegant ease with which the factorisation of amplitudes or form factors, i.e. the decoupling of soft and collinear degrees of freedom, can be formulated as consequence of simple field transformations of operators in the effective Lagrangian which decouple (ultra-)soft from collinear fields. Factorisation proofs at the Lagrangian level are automatically valid to all orders in perturbation theory and as such intriguing from a field-theoretical point of view, although the practitioner will probably be more grateful for explicit LO or NLO expressions. Much of this paper will deal with the discussion of the factorisation formula for one  $B \rightarrow \pi$  form factor derived in Ref. [14] and its detailed comparison with the corresponding QCD sum rule on the light-

cone. With SCET’s emphasis on factorisation, it is not surprising that the categories of “hard” and “soft” contributions which are appropriate and well-defined in LCSR, are less convenient in a SCET context and are replaced by “factorisable” and “nonfactorisable”. But also these categories, by themselves, are still subject to a certain arbitrariness as for the definition of the factorisation scheme. Nevertheless, to leading order in the heavy quark expansion, SCET spin-symmetry is effective and provides a natural factorisation scheme, with symmetry-breaking corrections induced by short-distance hard-gluon exchange and calculable in perturbation theory [10]. As it was shown in Ref. [12], and as I shall explicitly demonstrate in this paper, SCET spin-symmetry is broken by nonfactorisable terms at higher order in the heavy quark expansion, which limits the usefulness of symmetry-relations to relate experimentally measured form factors to unknown ones.

Let me summarise the questions to be addressed in this paper:

- What is the relation between factorisation formulas and expressions for form factors from QCD sum rules on the light-cone?
- How large are factorisable contributions to form factors as compared to nonfactorisable terms?
- How large are power-suppressed corrections to form factor relations?

The outline of the paper is as follows: in Sec. 2 I define the relevant form factors and give a short review of the basics of QCD sum rules on the light-cone as well as of  $\pi$  and  $B$  distribution amplitudes on the light-cone. In Sec. 3 I review the SCET factorised predictions for  $B \rightarrow \pi$  form factors and relate them to LCSR predictions. In Sec. 4 I discuss the numerics of factorised expressions and power-suppressed terms, and in Sec. 5 I present a summary and conclusions.

## 2 Definitions and Framework

$B \rightarrow \pi$  decays can be described by three form factors,  $f_+$ ,  $f_0$  and  $f_T$ , which are defined by ( $q = p_B - p$ ,  $q^2 = m_B^2 - 2m_B E_\pi$ )

$$\langle \pi(p) | \bar{u} \gamma_\mu b | B(p_B) \rangle = f_+(q^2) \left\{ (p_B + p)_\mu - \frac{m_B^2 - m_\pi^2}{q^2} q_\mu \right\} + \frac{m_B^2 - m_\pi^2}{q^2} f_0(q^2) q_\mu, \quad (1)$$

$$\begin{aligned} \langle \pi(p) | \bar{d} \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B(p_B) \rangle &\equiv \langle \pi(p) | \bar{d} \sigma_{\mu\nu} q^\nu b | B(p_B) \rangle \\ &= i \left\{ (p_B + p)_\mu q^2 - q_\mu (m_B^2 - m_\pi^2) \right\} \frac{f_T(q^2)}{m_B + m_\pi}. \end{aligned} \quad (2)$$

In the context of LCSR, it is convenient to introduce one more form factor:

$$f_-(q^2) = \frac{m_B^2 - q^2}{q^2} (f_0(q^2) - f_+(q^2)). \quad (3)$$

Note that  $f_+(0) = f_0(0)$ . For large energies of the final state  $\pi$ , i.e.  $E_\pi \sim O(m_b/2)$ , the form factors can be calculated from QCD sum rules on the light-cone. This method

combines standard QCD sum rule techniques with the information on light-cone hadron distribution amplitudes (DAs) familiar from the theory of exclusive processes. The key idea is to consider a correlation function of the weak current and a current with the quantum-numbers of the B meson, sandwiched between the vacuum and the  $\pi$ . For large (negative) virtualities of these currents, the correlation function is, in coordinate-space, dominated by light-like distances and can be expanded around the light-cone. In contrast to the short-distance expansion employed in conventional QCD sum rules à la SVZ [8], where nonperturbative effects are encoded in vacuum expectation values of local operators with vacuum quantum numbers, the condensates, LCSR $s$  rely on the factorisation of the underlying correlation function into genuinely nonperturbative and universal hadron DAs  $\phi$ . The DAs are convoluted with process-dependent amplitudes  $T_H$ , which are the analogues to Wilson-coefficients in short-distance expansion and can be calculated in perturbation theory, schematically

$$\text{correlation function} \sim \sum_n T_H^{(n)} \otimes \phi^{(n)}. \quad (4)$$

The sum runs over contributions with increasing twist, labelled by  $n$ , which are suppressed by increasing powers of, roughly speaking, the virtualities of the involved currents. The same correlation function can, on the other hand, be written as a dispersion-relation, in the virtuality of the current coupling to the B meson. Equating dispersion-representation and light-cone expansion, and separating the B meson contribution from that of higher one- and multi-particle states, one obtains a relation (QCD sum rule) for the form factor.

For  $B \rightarrow \pi$  form factors the relevant correlation function is

$$i \int d^4y e^{-ip_B y} \langle \pi(p) | T J_\mu(0) j_B^\dagger(y) | 0 \rangle = \Pi_+(p_B + p)_\mu + \Pi_-(p_B - p)_\mu \quad (5)$$

with  $j_B = \bar{d}i\gamma_5 b$  and  $J_\mu = \bar{u}\gamma_\mu b$  for  $f_{+,0}$  and  $J_\mu = \bar{d}\sigma_{\mu\nu}q^\nu b$  for  $f_T$ . LCSR $s$  for all three form factors are available at  $O(\alpha_s)$  accuracy for the twist-2 and tree-level accuracy for twist-3 and 4 contributions [15, 16]. Radiative corrections to the 2-particle twist-3 contributions to  $f_+$  have been calculated in Ref. [17]. For this paper, I have also calculated the corresponding corrections for  $f_-$  and  $f_T$ . The correlation function  $\Pi_+$ , calculated for unphysical  $p_B^2$ , can be written as dispersion-relation over its physical cut. Singling out the contribution of the B-meson, one has, for the vector current  $J_\mu = \bar{u}\gamma_\mu b$ ,

$$\Pi_+ = f_+(q^2) \frac{m_B^2 f_B}{m_B^2 - p_B^2} + \text{higher poles and cuts}, \quad (6)$$

where  $f_B$  is the leptonic decay constant of the B-meson,  $f_B m_B^2 = m_b \langle B | \bar{b}i\gamma_5 d | 0 \rangle$ . In the framework of LCSR $s$  one does not use (6) as it stands, but performs a Borel-transformation,

$$\hat{B} \frac{1}{t - p_B^2} = \frac{1}{M^2} \exp(-t/M^2), \quad (7)$$

with the Borel-parameter  $M^2$ ; this transformation enhances the ground-state B-meson contribution to the dispersion-representation of  $\Pi_+$  and suppresses contributions of higher twist to its light-cone expansion. The next step is to invoke quark-hadron duality to

approximate the contributions of hadrons other than the ground-state B-meson by the imaginary part of the light-cone expansion of  $\Pi_+$ , so that

$$\hat{B}\Pi_+^{\text{LC}} = \frac{1}{M^2} m_B^2 f_B f_+(q^2) e^{-m_B^2/M^2} + \frac{1}{M^2} \frac{1}{\pi} \int_{s_0}^{\infty} dt \text{Im}\Pi_+^{\text{LC}}(t) \exp(-t/M^2) \quad (8)$$

$$\text{and } \hat{B}_{\text{sub}}\Pi_+^{\text{LC}} = \frac{1}{M^2} m_B^2 f_B f_+(q^2) e^{-m_B^2/M^2}. \quad (9)$$

Eq. (9) is the LCSR for  $f_+$ .  $s_0$  is the so-called continuum threshold, which separates the ground-state from the continuum contribution. At tree-level, the continuum-subtraction in (9) introduces a lower limit of integration,  $u \geq (m_b^2 - q^2)/(s_0 - q^2) \equiv u_0$ , in (4), which behaves as  $1 - \Lambda_{\text{QCD}}/m_b$  for large  $m_b$  and thus corresponds to the dynamical configuration of the Feynman-mechanism, as it cuts off low momenta of the u quark created at the weak vertex. This is how soft contributions enter LCSRs. At  $\mathcal{O}(\alpha_s)$ , there are also contributions with no cut in the integration over  $u$ , which correspond to hard-gluon exchange contributions. As with standard QCD sum rules, the use of quark-hadron duality above  $s_0$  and the choice of  $s_0$  itself introduce a certain model-dependence (or systematic error) in the final result for the form-factor.

Let me shortly comment on the expansion parameter of LCSRs. It has been claimed that the expansion goes in powers of  $1/m_b$ , e.g. [18]. The situation is not that simple, however. In the next section we shall see that twist-3 DAs contribute at the same order in  $1/m_b$  as twist-2 ones – this observation is not really new and has first been made in Ref. [19]. It actually turns out that the coefficients of twist-3 and 4 amplitudes, as far as they are known, do not show any suppression by powers of  $m_b$  at all, but that it is the shape of distribution amplitudes as predicted by conformal expansion that entails an effective suppression in inverse powers of  $m_b$  upon (soft) convolution over  $u$ . This suppression becomes effective only if an *explicit*  $1/m_b$  expansion is performed (as we shall do in the next section). In addition, tree-level power-counting may be upset by radiative corrections: it turns out that the tree-level 3-particle twist-3 contribution is suppressed by  $1/m_b$  with respect to the twist-2 contribution, but this suppression is not removed by  $\mathcal{O}(\alpha_s)$  corrections. In general, for  $\pi$  DAs, we expect that higher twist DAs are dominated by *gluonic* DAs, and that their matrix elements are suppressed numerically (not parametrically) by powers of  $\alpha_s$ .

To twist-3 accuracy, the DAs of the  $\pi$  are defined as ( $\bar{u} = 1 - u$ )

$$\langle \pi | \bar{u}(0) \gamma_\mu \gamma_5 [0, x] u(x) | 0 \rangle = -i p_\mu f_\pi \int du e^{i \bar{u} p x} \phi_\pi(u), \quad (10)$$

$$\langle \pi | \bar{u}(0) i \gamma_5 [0, x] d(x) | 0 \rangle = \mu_\pi^2 \int du e^{i \bar{u} p x} \phi_p(u), \quad (11)$$

$$\langle \pi | \bar{u}(0) \sigma_{\alpha\beta} i \gamma_5 [0, x] d(x) | 0 \rangle = \frac{1}{6} \mu_\pi^2 (p_\alpha x_\beta - x_\alpha p_\beta) \int du e^{i \bar{u} p x} \phi_\sigma(u),$$

$$\begin{aligned} & \langle \pi | \bar{u}(0) \sigma_{\alpha\beta} \gamma_5 [0, vx] g G_{\mu\nu}(vx) [vx, x] d(x) | 0 \rangle = \\ & = i \mu_\pi^2 [(p_\mu p_\alpha g_{\nu\beta} - (\mu \leftrightarrow \nu)) - (p_\mu p_\beta g_{\nu\alpha} - (\mu \leftrightarrow \nu))] \int \mathcal{D}\alpha \mathcal{T}(\underline{\alpha}) e^{ipx(\alpha_2 + v\alpha_3)}. \end{aligned} \quad (12)$$

Here  $[x, y]$  is the path-ordered gauge-factor

$$[x, y] = P \exp \left[ ig \int_0^1 dt (x - y)_\mu A^\mu (tx + (1-t)y) \right],$$

$\mu_\pi^2 = f_\pi m_\pi^2 / (m_u + m_d) = -2\langle \bar{q}q \rangle / f_\pi$  and  $\mathcal{D}\alpha = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$ .  $\phi_\pi$  is a twist-2 DA,  $\phi_\sigma$ ,  $\phi_p$ ,  $\mathcal{T}$  are, loosely speaking, twist-3, although  $\phi_\sigma$  and  $\phi_p$  also contain admixtures of twist-2 contributions. The DAs can be expressed as a partial wave expansion in conformal spin (conformal expansion); more details and explicit expressions can be found in Ref. [20]. For the present study, we mostly need the so-called asymptotic DAs, the leading term in the partial wave expansion:

$$\phi_\pi(u) = \phi_\sigma(u) = 6u(1-u), \quad \phi_p(u) = 1, \quad \mathcal{T}(\underline{\alpha}) = 360\eta_3(\mu)\alpha_1\alpha_2\alpha_3^2, \quad (13)$$

where the hadronic matrix element  $\eta_3(\mu)$  is defined by the local limit of (12). To higher order in the expansion, also the 2-particle DAs become scale-dependent and observe evolution equations. We will need in particular the evolution equation of  $\phi'_\pi(1)$ , first derived in [16] ( $a_s = C_F\alpha_s/(4\pi)$ ):

$$\mu \frac{d}{d\mu} \phi'_\pi(1, \mu) = -4a_s \left\{ \int_0^1 du \left[ \frac{\phi_\pi(u) + \bar{u}\phi'_\pi(1)}{\bar{u}^2} + \frac{\phi_\pi(u)}{\bar{u}} \right] - \frac{1}{2} \phi'_\pi(1) \right\}. \quad (14)$$

Note that the three twist-3 DAs are not independent of each other. Solving the recursion relation for the moments of e.g.  $\phi_p(u)$ , given in [20], I find ( $\xi = 2u - 1$ ):

$$\frac{d}{du} \phi_p(u) = \frac{1}{1 - \xi^2} \left( \xi \frac{d^2}{du^2} \phi_{\mathcal{T}_1}(u) - 2 \frac{d}{du} \phi_{\mathcal{T}_1}(u) + \frac{d^2}{du^2} \phi_{\mathcal{T}_2}(u) \right), \quad (15)$$

$$\text{with } \phi_{\mathcal{T}_1}(u) = \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \frac{2}{\alpha_3} \mathcal{T}(\underline{\alpha}), \quad \phi_{\mathcal{T}_2}(u) = \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_2 \frac{2(\alpha_1 - \alpha_2 - \xi)}{\alpha_3^2} \mathcal{T}(\underline{\alpha}).$$

The initial condition for the differential equation (15) is given by the recursion relation for the second moment of  $\phi_p$ , given in [20]. Solving (15), I find:

$$\phi_p(u, \mu) = 1 + 12\eta_3(\mu) + \frac{1}{2} \int_0^1 dv (2v - 1)^3 \frac{d}{dv} \phi_p(v, \mu) + \int_0^u dv \frac{d}{dv} \phi_p(v, \mu), \quad (16)$$

which expresses  $\phi_p$  uniquely in terms of the 3-particle DA  $\mathcal{T}$ . A similar relation can be derived for  $\phi_\sigma$ . The evolution equation for  $\mathcal{T}$  is not known.

For completeness, let me also introduce the leading-twist B meson DA which features in SCET factorised expressions (but not in LCSR). It can be defined as<sup>1</sup> [21]

$$\langle 0 | \bar{q}_s(x) [x, 0]_s \not{p} \Gamma h(0) | B(v) \rangle = -i \frac{F(\mu)}{2} \text{Tr}(\not{p} \Gamma \frac{1 + \not{v}}{2} \gamma_5) \int_0^\infty d\omega e^{-i\omega(v \cdot x)} \phi_+^B(\omega, \mu), \quad (17)$$

where  $n$  is a light-like vector parallel to  $x$  and  $F(\mu)$  is the matrix element corresponding to the asymptotic value of  $\sqrt{m_B}f_B$  in heavy quark effective theory (HQET);  $v$  is the four-velocity of the B meson. The index  $s$  denotes the soft modes of (quark and gluon) fields

<sup>1</sup>To be precise, there are *two* B DAs, but only one of them enters the leading-order SCET factorised expressions for  $B \rightarrow \pi$  form factors.

in SCET and  $h$  is the heavy quark field in HQET. The normalisation of  $\phi_+^B$  is chosen such that

$$\int_0^\infty d\omega \phi_+^B(\omega, \mu) = 1.$$

The scale-dependence of  $\mu$  has been discussed in [21], with the result that running from an initial scale  $\mu_0$  to  $\mu$  induces a radiative tail of the DA that falls off slower than  $1/\omega$  at large  $\omega$ , which renders positive moments of the DA ill-defined, independent of its behaviour at  $\mu_0$ . In SCET factorised expressions, however,  $\phi_+^B(\omega)$  always enters in the combination  $\phi_+^B(\omega)/\omega$ , modulo logs, i.e. as

$$\int_0^\infty d\omega \frac{\phi_+^B(\omega)}{\omega} \ln^n \frac{\omega}{\mu},$$

which is well-defined at all scales. For  $n = 0$ , a simple evolution equation emerges. Defining

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty d\omega \frac{\phi_+^B(\omega, \mu)}{\omega},$$

I extract the following evolution equation from the expressions given in Ref. [21]:

$$\mu \frac{d}{d\mu} \frac{1}{\lambda_B(\mu)} = 2a_s \frac{1}{\lambda_B(\mu)} + 4a_s \int_0^\infty d\omega \frac{\phi_+^B(\omega, \mu)}{\omega} \ln \frac{\omega}{\mu}. \quad (18)$$

I have checked that the above evolution equation indeed ensures  $\mu$ -independence of the factorised expression for the  $B \rightarrow \gamma$  form factor obtained in Ref. [22].

### 3 Light-Cone Sum Rules vs. SCET – A Case Study

In the SCET limit  $m_b \rightarrow \infty$ ,  $E_\pi \rightarrow \infty$ , spin-symmetry ensures that the three  $B \rightarrow \pi$  form factors can be expressed in terms of one single nonperturbative function  $\zeta(E_\pi)$  that includes the soft contributions. Spin-symmetry is broken by two effects: hard gluonic corrections to the weak vertex, which yield vertex-specific matching conditions at  $\mu = m_b$ , whereas the logarithmic terms  $\ln(m_b/\mu)$  are spin-symmetric, and hard spectator interactions.

Combining the results of Refs. [10] and [11], the form factors at  $q^2 = 0$ , i.e.  $E_\pi = m_B/2 = E_{\max}$  can be written as ( $a_s = C_F \alpha_s / (4\pi)$ ):

$$f_+(0) = \left[ C_4 \left( \frac{m_b}{\mu_F} \right) + \frac{1}{2} C_5 \left( \frac{m_b}{\mu_F} \right) \right] \zeta(E_{\max}, \mu_F) + a_s \Delta f_+(\mu_F) + O(1/m_b), \quad (19)$$

$$f_-(0) = - \left[ C_4 \left( \frac{m_b}{\mu_F} \right) - \frac{1}{2} C_5 \left( \frac{m_b}{\mu_F} \right) \right] \zeta(E_{\max}, \mu_F) + a_s \Delta f_-(\mu_F) + O(1/m_b), \quad (20)$$

$$f_T(0, \mu) = C_T \left( \frac{m_b}{\mu} \right) \left[ C_{11} \left( \frac{m_b}{\mu_F} \right) \zeta(E_{\max}, \mu_F) + a_s \Delta f_T(\mu_F) \right] + O(1/m_b). \quad (21)$$

The phenomenological interest of factorised expressions like the above lies in the fact that the nonperturbative quantity  $\zeta$  can be eliminated in ratios of form factors, which are then predicted in terms of calculable expressions. The usefulness of such relations depends on

the size of neglected terms; we shall come back to that question in Sec. 4.  $C_T$  is the standard QCD Wilson-coefficient for the tensor-current, whereas  $C_i$  resums large single and double logarithms  $\sim \ln m_b/\mu_F$  encountered in the matching of SCET<sub>I</sub> onto QCD. Explicit resummed expressions can be found in [14]; expanded to first order in  $\alpha_s$ , one has

$$\begin{aligned} C_4 \left( \frac{m_b}{\mu_F} \right) &= \left\{ 1 - a_s \left( \frac{\pi^2}{12} + \frac{13}{2} \right) \right\} \left\{ 1 + a_s \left( -2 \ln^2 \frac{m_b}{\mu_F} + 5 \ln \frac{m_b}{\mu_F} \right) \right\}, \\ C_5 \left( \frac{m_b}{\mu_F} \right) &= a_s, \\ C_{11} \left( \frac{m_b}{\mu_F} \right) &= \left\{ 1 - a_s \left( \frac{\pi^2}{12} + 4 \right) \right\} \left\{ 1 + a_s \left( -2 \ln^2 \frac{m_b}{\mu_F} + 5 \ln \frac{m_b}{\mu_F} \right) \right\}, \\ C_T \left( \frac{m_b}{\mu} \right) &= 1 + a_s \ln \frac{m_b^2}{\mu^2}. \end{aligned} \quad (22)$$

In the notation of Ref. [10], the  $\Delta f$  in Eqs. (19) to (21) denote the contributions from hard-gluon exchange between the b or u quark (d quark for  $f_T$ ) and the spectator quark. The corresponding diagrams contain endpoint singularities, but the authors of Ref. [10] argued that these singularities are spin-symmetric and hence can be absorbed into  $\zeta$ . From this it is evident that the separation between ‘‘hard’’ and ‘‘soft’’ contributions is not well-defined *eo ipso*, but requires the definition of a factorisation scheme that fixes precisely which terms are included into  $\zeta$  and which into  $\Delta f$ . In Ref. [10], the factorisation scheme was defined by

$$f_+(q^2) \equiv \xi(E_\pi).$$

In this scheme also hard-gluon corrections to the weak vertex which contain double logarithms, but are spin-symmetric, are absorbed into  $\xi$ . To  $O(\alpha_s)$  accuracy, the following relations were obtained:

$$f_-(0) = -\xi(E_{\max}) \{1 - a_s\} + a_s \Delta F \quad (23)$$

$$f_T(0, \mu) = \frac{m_B + m_\pi}{m_B} \xi(E_{\max}) \left\{ 1 + a_s \left( \ln \frac{m_b^2}{\mu^2} + 2 \right) \right\} - a_s \frac{m_B + m_\pi}{m_B} \Delta F, \quad (24)$$

$$\text{with } \Delta F = \frac{8\pi^2}{3} \frac{f_B f_\pi}{m_B} \frac{1}{\lambda_B} \left\langle \frac{1}{\bar{u}} \right\rangle_\pi. \quad (25)$$

The results of Ref. [10] indicate that the classification of form factor contributions as ‘‘hard’’ or ‘‘soft’’ may not be the most appropriate one for heavy-to-light form factors and should be replaced by one distinguishing between spin-symmetric and non-symmetric terms. This is indeed the pattern emerging in SCET, where the following factorisation formula, valid to all orders in  $\alpha_s$  and leading order in  $1/m_b$ , has been derived in Ref. [14]:

$$f_+(q^2) = \left( C_4(E_\pi, m_b/\mu_I) + \frac{E_\pi}{m_B} C_5(E_\pi, m_b/\mu_I) \right) \zeta(E_\pi, \mu_I)$$

$$\begin{aligned}
& + \frac{m_B f_B f_\pi}{4E_\pi^2} \int_0^\infty dk_+ \int_0^1 du dv \phi_B^+(k_+, \mu_{II}) \phi_\pi(u, \mu_{II}) \\
& \times \left( \frac{2E_\pi - m_B}{m_B} T_a(\mu_I) J_a(u, v, k_+; \mu_I, \mu_{II}) + \frac{2E_\pi}{m_B} T_b(\mu_I) J_b(u, v, k_+; \mu_I, \mu_{II}) \right). \quad (26)
\end{aligned}$$

Here  $\mu_{I(II)}$  is a scale in SCET<sub>I(II)</sub>, i.e.  $\Lambda_{\text{QCD}} \lesssim \mu_{II} < \sqrt{m_b \Lambda_{\text{QCD}}} \lesssim \mu_I < m_b$ .  $T_i$  are SCET<sub>I</sub> Wilson-coefficients of (subleading) currents in the SCET<sub>I</sub> Lagrangian and the jet functions  $J_i$  are the Wilson-coefficients of matching SCET<sub>II</sub> quark bilinear operators onto SCET<sub>I</sub>, with, to  $O(\alpha_s)$  accuracy [14]:

$$T_a J_a = T_b J_b = \frac{4\pi^2}{3} a_s \frac{\delta(u - v)}{u k_+}.$$

The first term in (26) is nonfactorisable, but spin-symmetric, except for the values of the matching coefficients at  $\mu_I = m_b$ . The second term is factorisable and does not contain endpoint singularities, but breaks spin-symmetry. One important consequence of the decoupling of collinear and soft degrees of freedom in SCET is that the two terms do not mix under a change of scales. As we shall see below, this suggests that all twist-3 effects which are of the same order in  $1/m_b$  as the twist-2 contributions can be absorbed into  $\zeta$ . Note that  $\zeta(E_\pi, \mu_F)$  is defined in SCET<sub>I</sub> and that also contributions of the second B DA  $\phi_B^-$  have been absorbed into it. The question arises if the fact that spin-symmetric and nonfactorisable terms coincide is an artifact of the leading order in the  $1/m_b$  expansion. As shown in Ref. [12] this is indeed the case and may limit the relevance of Eq. (26) from a phenomenological point of view, as nonfactorisable (uncalculable) symmetry-breaking terms at nonleading order in  $1/m_b$  cannot be eliminated any more from form factor relations.

In the remainder of this section, I will compare the LCSR predictions for  $f_{+,-,T}$  with the above relations and address in particular the following questions:

- do LCSRs reproduce the relations (23), (24) and (25)?
- do LCSRs reproduce the logarithmic terms resummed in the Wilson-coefficients  $C_i$ ?
- can (26) be directly derived from LCSRs (to  $O(\alpha_s)$ )?
- are there soft symmetry-breaking contributions at nonleading order in  $1/m_b$ ?

In order to compare light-cone sum rules with the factorisation formulas predicted by SCET, I have

- calculated the LCSRs for all three form factors for  $B \rightarrow \pi$  transitions in full QCD, at  $q^2 = 0$ , i.e.  $E_\pi = m_B/2$ , to  $O(\alpha_s)$  accuracy for 2-particle twist-2 and 3 contributions;
- performed the heavy quark limit in the following way [16]: the sum rule parameters  $M^2$  and  $s_0$  are scaled with the heavy quark mass as

$$M^2 = 2m_b \tau, \quad s_0 - m_b^2 = 2m_b \omega_0, \quad (27)$$

with  $\tau$  and  $\omega_0 \sim 1 \text{ GeV}$ . These scaling laws follow from the requirement of the sum rule for  $f_B$  to observe the correct scaling in the heavy quark limit [23]. In addition,

in order to obtain simple expressions, I also fix  $q^2 = 0$  ( $E_\pi = m_B/2$ ) and apply the finite-energy limit  $\tau \rightarrow \infty$ . In this limit, the B meson corresponds to a heavy quark accompanied by light degrees of freedom with an energy less or equal  $\omega_0$ , i.e. a distribution with a sharp cutoff.

The rationale behind this procedure is the observation that SCET factorisation formulas rely on manipulations of the Lagrangian and hence are independent of the realisation of physical states; consequently, they must also be realised in LCSR.

In the combined heavy quark and finite-energy limit LCSR depend on the  $\pi$  DAs of leading and higher twist,  $\omega_0$ , the only parameter characterising the B meson, the factorisation scale  $\mu$  characteristic for the  $\pi$ , and  $m_b$ . For  $f_+$  and twist-2 accuracy, the corresponding formula was first obtained in [16]. Radiative corrections to the twist-3 contributions to  $f_+$  have been calculated in [17]. Here, we extend these analyses and include also radiative corrections to twist-2 and twist-3 2-particle DAs for  $f_-$  and  $f_T$ . The corresponding expressions are given in the appendix. The contributions to  $f_+(0)$  read:

$$f_B m_b f_+^{T2}(0) = -f_\pi \frac{\omega_0^2}{m_b} \phi'_\pi(1, \mu) \left\{ 1 + a_s \left( 1 + \pi^2 - 2 \ln^2 \frac{2\omega_0}{m_b} - 4 \ln \frac{2\omega_0}{m_b} + 2 \ln \frac{2\omega_0}{\mu} \right) \right\} + 4a_s f_\pi \frac{\omega_0^2}{m_b} \left\{ \left( \ln \frac{2\omega_0}{\mu} - 1 \right) \int_0^1 du \frac{\phi_\pi(u) + \bar{u}\phi'_\pi(1)}{\bar{u}^2} + \ln \frac{2\omega_0}{\mu} \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} \right\}, \quad (28)$$

$$f_B m_b f_+^{T3,p}(0) = \mu_\pi^2(\mu) \frac{\omega_0}{m_b} \phi_p(1, \mu) \left\{ 1 + a_s \left( \pi^2 - 4 - 2 \ln^2 \frac{2\omega_0}{m_b} + 4 \ln \frac{2\omega_0}{\mu} - 4 \ln \frac{2\omega_0}{\mu} \ln \frac{2\omega_0}{m_b} \right) \right\} + 2\mu_\pi^2 \frac{\omega_0}{m_b} a_s \left( 4 \ln \frac{2\omega_0}{\mu} - 3 \right) \int_0^1 du \frac{1}{\bar{u}} (\phi_p(u) - \phi_p(1)), \quad (29)$$

$$f_B m_b f_+^{T3,\sigma}(0) = -\mu_\pi^2(\mu) \frac{\omega_0}{6m_b} \phi'_\sigma(1, \mu) \left\{ 1 + a_s \left( \pi^2 - 10 - 2 \ln^2 \frac{2\omega_0}{m_b} - 8 \ln \frac{2\omega_0}{m_b} + 8 \ln \frac{2\omega_0}{\mu} + 4 \ln \frac{2\omega_0}{\mu} \ln \frac{2\omega_0}{m_b} \right) \right\} - \mu_\pi^2 \frac{\omega_0}{3m_b} a_s \int_0^1 du \frac{1}{\bar{u}^2} (\phi_\sigma(u) + \bar{u}\phi'_\sigma(1)). \quad (30)$$

Let me first discuss  $f_+^{T2}(0)$ . The first point to notice is that by virtue of the evolution equation (14), the r.h.s. of (28) is indeed independent of the factorisation scale  $\mu$ . Soft and hard contributions can clearly be identified: soft terms are those with a highly asymmetric configuration of the quarks in the  $\pi$ , i.e. the terms in  $\phi'_\pi(1)$ , whereas those in  $\int_0^1 du$  are hard contributions. Note that both contributions combine in such a way that integrands with  $\bar{u}^2$  in the denominator that could give rise to endpoint singularities are automatically regularised. It is also evident that the separation between hard and soft contributions is  $\mu$ -dependent.

Let us next look at the logarithms in  $m_b/\mu$ . It is clear that  $2\omega_0$  has to be identified with  $\mu_{II}$ , a scale in SCET<sub>II</sub>. It is thus not to be expected that (28) exactly reproduces the logarithms resummed in the Wilson-coefficients  $C_i$ , Eq. (22), which are characteristic for SCET<sub>I</sub>. Indeed, subtracting  $C_4(m_b/(2\omega_0))[1 + 3a_s \ln(m_b/(2\omega_0))]$  from (28) (the second

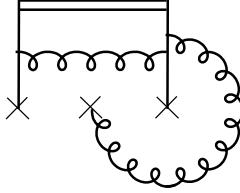


Figure 2:  $O(\alpha_s)$  contribution of  $\mathcal{T}$  to  $f_+$ . Double line: b quark propagator, lines ending with a cross: onshell partons.

logarithmic term accounts for the  $\mu$ -dependence of  $f_B \sqrt{m_B}$  in the HQL), the double logs cancel, but a term  $-4 \ln m_b / (2\omega_0)$  remains. This term is in fact universal for all form factors contributions, including those listed in the appendix. This suggests that they are related to matching effects of SCET<sub>II</sub> onto SCET<sub>I</sub> which are not included in (28).

As for the contributions to  $f_+$  induced by the twist-3 DAs  $\phi_p$  and  $\phi_\sigma$ , at first glance they do not appear to follow the pattern displayed by  $f_+^{T2}$ : there are mixed logarithms  $\sim \ln(2\omega_0/\mu) \ln(2\omega_0/m_b)$ . These logarithms disappear upon implementing the equation of motion constraint  $\phi'_\sigma(1, \mu) = -6\phi_p(1, \mu)$ , which follows from the conformal expansion of the DAs discussed in Ref. [20] and is valid exactly (i.e. to all orders in the conformal expansion). There is a similar relation for  $u = 0$ :  $6\phi_p(0, \mu) = \phi'_\sigma(0, \mu)$ . One then has

$$\begin{aligned}
f_B m_b f_+^{T3}(0) &= f_B m_b (f_+^{T3,p}(0) + f_+^{T3,\sigma}(0)) \\
&= 2\mu_\pi^2(\mu) \frac{\omega_0}{m_b} \phi_p(1, \mu) \left\{ 1 + a_s \left( \pi^2 - 7 - 2 \ln^2 \frac{2\omega_0}{m_b} - 4 \ln \frac{2\omega_0}{m_b} + 6 \ln \frac{2\omega_0}{\mu} \right) \right\} \\
&\quad + 2\mu_\pi^2(\mu) \frac{\omega_0}{m_b} a_s \left( 4 \ln \frac{2\omega_0}{\mu} - 3 \right) \int_0^1 du \frac{\phi_p(u) - \phi_p(1)}{\bar{u}} \\
&\quad - \mu_\pi^2(\mu) \frac{\omega_0}{3m_b} a_s \int_0^1 du \frac{\phi_\sigma(u) + \bar{u}\phi'_\sigma(1)}{\bar{u}^2}.
\end{aligned} \tag{31}$$

The term in  $6 \ln(2\omega_0/\mu)$  cancels the  $\mu$ -dependence of  $\mu_\pi^2$ . As for the remaining logarithms in  $2\omega_0/\mu$ , they contribute only at nonleading order in the conformal expansion of  $\phi_p$  and do not cancel the  $\mu$ -dependence of  $\phi_p(1, \mu)$ . This is to be expected as, as discussed in Sec. 2,  $\phi_p$  mixes with the 3-particle DA  $\mathcal{T}$  and we expect only the combination  $f_B m_b (f_+^{T3,p} + f_+^{T3,\sigma} + f_+^{T3,\mathcal{T}})$  to be  $\mu$ -independent. At first glance, this appears to be in contradiction with the HQL of the tree-level contribution of  $\mathcal{T}$ , which is suppressed by one power of  $m_b$ , which indeed has caused some confusion, cf. [18]. The power-suppression of the tree-diagram is however a consequence of the fact that the gluon line is emitted from the b quark and is expected to be removed at  $O(\alpha_s)$ , as the complete expression  $f_+^{T3}$  must be  $\mu$ -independent. To verify this expectation, I have calculated the diagram shown in Fig. 2. In the HQL and in Feynman gauge, it yields (using the asymptotic form of  $\mathcal{T}$ )

$$f_B m_b f_+^{\text{Fig. 2}}(0) = 2\mu_\pi^2 \frac{\omega_0}{m_b} a_s (-160\eta_3), \tag{32}$$

i.e. the power-suppression effective at tree-level is indeed removed. Numerically,  $\eta_3(1 \text{ GeV}) \approx 0.01$  [20]. The reason for the huge factor 160 is the factor 360 in the asymptotic DA,

Eq. (13). The size of this contribution has to be compared with the one induced by the integral over  $\phi_p$  in (31), which gives 270, in the same units. Assuming that the diagram in Fig. 2 yields a typical contribution, the  $O(\alpha_s)$  corrections to  $\mathcal{T}$  should turn out to be of roughly the same size as the  $O(\alpha_s)$  contributions induced by  $\phi_p$  and  $\phi_\sigma$ , which, in full LCSR without the finite-energy limit  $M^2 \rightarrow \infty$  are nonnegligible, but not sizeable [17].

Using the formulas collected in App. A, it is straightforward to verify that LCSR also reproduce the structure of symmetry-breaking hard vertex-corrections

$$\begin{aligned} f_+(0) + f_-(0) &= a_s \xi(E_{\max}) + \dots \\ f_T(0, \mu) - f_+(0) &= a_s \xi(E_{\max}) \left( 2 + \ln \frac{m_b^2}{\mu^2} \right) + \dots \end{aligned} \quad (33)$$

with

$$\xi(E_{\max}) = \frac{1}{f_B m_b} \left( -f_\pi \frac{\omega_0^2}{m_b} \phi'_\pi(1) + 2\mu_\pi^2 \frac{\omega_0}{m_b} \phi_p(1) + O(\alpha_s) \right),$$

where the dots denote integral terms  $\sim \int_0^1 du$ .

The last term in (23) and (24) that has to be identified with a corresponding structure in the LCSR is  $\Delta F$ , Eq. (25). In order to do so, one has to express  $1/\lambda_B$  in terms of a finite-energy sum rule. Such a sum rule has been obtained in Ref. [24] from a comparison of the SCET factorised expression for the  $B \rightarrow \gamma$  form factor and its SVZ sum rule and reads:

$$f_B^2 m_b \frac{1}{\lambda_B} = \frac{3}{2\pi^2} \omega_0^2. \quad (34)$$

$\Delta F$  should thus translate into

$$f_B m_b \Delta F = 4f_\pi \frac{\omega_0^2}{m_b} \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}}.$$

Indeed, I find

$$\begin{aligned} f_B m_b (f_T^{T2}(0) - f_+^{T2}(0)) &= -4f_\pi a_s \frac{\omega_0^2}{m_b} \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} + \text{terms with no integral,} \\ f_B m_b (f_T^{T3}(0) - f_+^{T3}(0)) &= 0 + \text{terms with no integral,} \\ f_B m_b (f_+^{T2}(0) + f_-^{T2}(0)) &= 4f_\pi a_s \frac{\omega_0^2}{m_b} \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} + \text{terms with no integral,} \\ f_B m_b (f_+^{T3}(0) + f_-^{T3}(0)) &= 0 + \text{terms with no integral.} \end{aligned} \quad (35)$$

This pattern of integral terms coincides with the SCET results (23) and (24) and implies that to leading order in  $1/m_b$  symmetry-breaking hard spectator interactions indeed involve only the twist-2 DA  $\phi_\pi$ . Although an explicit calculation of  $O(\alpha_s)$  contributions to  $\mathcal{T}$  is not available, we do not expect them to break spin-symmetry: the three twist-3 DAs  $\phi_p$ ,  $\phi_\sigma$  and  $\mathcal{T}$  are coupled by evolution equations, so the presence of symmetry-breaking contributions from  $\mathcal{T}$  would entail corresponding contributions from  $\phi_p$  and  $\phi_\sigma$ . The only way this could be avoided is if  $\mathcal{T}$  observes a evolution equation of its own, i.e. if it mixes into  $\phi_p$  and  $\phi_\sigma$ , but not vice versa.

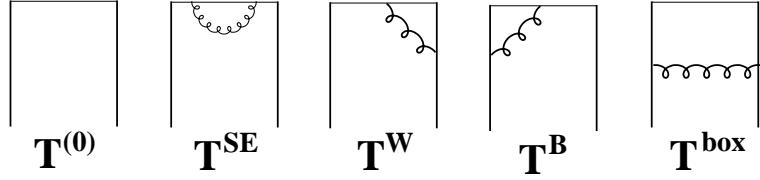


Figure 3: Radiative corrections to the correlation function (5). The external quark lines are onshell with momenta  $up$  and  $\bar{u}p$ , respectively.  $T^B$ , the radiative correction to the B vertex, and  $T^{\text{box}}$  correspond to hard spectator interaction diagrams in SCET.

So far we have essentially verified that LCSR reproduce the relations for the differences of form factors derived in Ref. [10]. The next question to ask is if the symmetry-breaking factorisable term in (26) can also be derived from LCSR. To this end, we look more closely at the calculation of radiative corrections to the correlation function (5). The one-loop Feynman diagrams are shown in Fig. 3. It turns out that to twist-2 accuracy they can be expressed in terms of a few basic traces:

$$\begin{aligned}
Tr_1 &= Tr(\mathcal{P}\Gamma q\gamma_5) \equiv Tr(\mathcal{P}\Gamma \not{p}_B \gamma_5), \\
Tr_2 &= Tr(\mathcal{P}\Gamma \gamma_5), \\
Tr_3 &= Tr(\mathcal{P}q\Gamma \gamma_5) \equiv Tr(\mathcal{P}\not{p}_B \Gamma \gamma_5), \\
Tr_4 &= Tr(\mathcal{P}q\Gamma q\gamma_5).
\end{aligned} \tag{36}$$

$\mathcal{P}$  is the projector onto the final state meson,  $\mathcal{P}_\pi = \not{p}\gamma_5$ , and  $\Gamma$  is the Dirac structure of the weak vertex. The contribution of each diagram to the correlation function is given by

$$\frac{i}{4} f_\pi \int_0^1 du \phi_\pi(u) T(u; p_B^2, q^2).$$

The tree-level diagram yields

$$T^{(0)} = \frac{i}{s} (Tr_1 + m_b Tr_2),$$

with  $s = m_b^2 - up_B^2 - \bar{u}q^2$ . The analogues of the hard spectators diagrams are  $T^B$  and  $T^{\text{box}}$ ; they are given by

$$\begin{aligned}
T^B &= 2 \frac{g^2 C_F}{s} \left\{ \left( -8\bar{C}(1+\epsilon) - 1 - m_b^2 \bar{B} + \bar{u}(p_B^2 - q^2) \bar{A} \right) (Tr_1 + m_b Tr_2) - m_b s \bar{B} Tr_2 \right\}, \\
T^{\text{box}} &= -g^2 C_F a_{\mathcal{P}} \left\{ a_{\mathcal{P}} H(Tr_1 + m_b Tr_2) + I(-m_b Tr_4 + (s - m_b^2) Tr_3) + \tilde{B} Tr_3 \right\}
\end{aligned} \tag{37}$$

with

$$a_{\mathcal{P}} \mathcal{P} := \gamma_\alpha \mathcal{P} \gamma^\alpha \quad \text{and} \quad \tilde{B} = B(u \rightarrow 1).$$

The integrals are defined as

$$\int \frac{d^D k}{(2\pi)^D} \frac{k_\alpha}{(k + up)^2 k^2 [(q - k)^2 - m^2]} = A u p_\alpha + B q_\alpha,$$

$$\int \frac{d^D k}{(2\pi)^D} \frac{k_\alpha}{(k - \bar{u}p)^2 k^2 [(p_B + k)^2 - m^2]} = -\bar{A}\bar{u}p_\alpha - \bar{B}p_{B\alpha},$$

$$\int \frac{d^D k}{(2\pi)^D} \frac{k_\alpha k_\beta}{k^2 (k - up)^2 (k + \bar{u}p)^2 [(up + q - k)^2 - m^2]} = Hg_{\alpha\beta} + Ig_\alpha q_\beta + \dots,$$

where the dots stand for irrelevant terms. All infrared divergent terms are treated in dimensional regularisation and can be absorbed into the distribution amplitude. All occurring convolution integrals are finite; there are no endpoint singularities.

The terms in  $Tr_1 + m_b Tr_2$  are obviously spin-symmetric; for  $f_+(0)$ , one has  $Tr_1 = Tr_3 = Tr_4 = 0$  and  $Tr_2 = -2$ . The only spin-symmetry breaking contribution to the correlation function  $\Pi_+$  is then

$$\Pi_+^{\text{sym-break}} = if_\pi a_s m_b (4\pi)^2 \int_0^1 du \phi_\pi(u) \bar{B}(u, p_B^2). \quad (38)$$

As shown in App. B, this expression yields

$$f_B m_b f_+^{\text{sym-break}}(0) = 2a_s f_\pi \frac{\omega_0^2}{m_b} \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} = f_B m_b \left( \frac{4\pi^2}{3} a_s \frac{f_B f_\pi}{m_b} \frac{1}{\lambda_B} \left\langle \frac{1}{\bar{u}} \right\rangle_\pi \right), \quad (39)$$

which coincides with the r.h.s. of (26) for  $E_\pi = m_B/2$  and to  $O(\alpha_s)$  accuracy.<sup>2</sup>

The last point on the list of items on p. 9 still to be investigated is the possible emergence of symmetry-breaking nonfactorisable terms at nonleading order in the heavy quark expansion. Such terms have been found in an expansion of the SCET Lagrangian to second order [12], and we find them also in the sum rule approach. Restricting ourselves to tree-level, we have

$$f_B m_B (f_T^{T3}(0) - f_+^{T3}(0)) = -4\mu_\pi^2 \frac{\omega_0^2}{m_b^2} \phi_p(1). \quad (40)$$

This evidently soft symmetry-breaking terms confirms the conclusion of [12] that simple symmetry-relations for form factors are broken by nonfactorisable power-suppressed corrections. We will discuss the numerical impact of these terms in the next section.

To summarise, I have found that, to  $O(\alpha_s)$  and twist-3 accuracy, and in the heavy quark limit,

- LCSR for the form factors of  $B \rightarrow \pi$  transitions observe the relations for spin-symmetry breaking corrections obtained in Ref. [10];
- LCSR predict the same symmetry-breaking hard-scattering amplitude as the SCET factorisation theorem [14];
- at  $E_\pi = m_b/2$ , LCSR reproduce the structure of double logarithms in  $m_b$ , but not of single logs, which I attribute to yet unknown matching effects from SCET<sub>II</sub> onto SCET<sub>I</sub>;

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<sup>2</sup>Up to the replacement  $u \leftrightarrow \bar{u}$  under the integral, which is justified for  $\phi_\pi$ , but not for  $\phi_K$ .

- the  $1/m_b$  suppression of 3-particle twist-3 contributions at  $O(1)$  is removed at  $O(\alpha_s)$ , as required by the  $\mu$ -independence of  $f_+$ ; as all effects of 2-particle twist-3 DAs are symmetry-preserving to  $O(\alpha_s)$ , and 2 and 3-particle DAs are linked by evolution equations, the same is likely to happen also for the 3-particle ones, so that it is expected that all twist-3 effects can be absorbed into  $\zeta$ ;
- LCSR $s$  are free of endpoint-singularities throughout, including the sample diagram Fig. 2; it should be possible to prove that within SCET itself, along precisely the same lines as other factorisation proofs.

At the end of this lengthy section, let me also state clearly where LCSR $s$  fall behind SCET factorised expressions: the resummation of large logarithms appears difficult, in particular since in the finite-energy limit there is no obvious candidate for the intermediate scale  $\sim \sqrt{m_b \Lambda_{\text{QCD}}}$ . It is possible that in full sum rules the Borel parameter may play that part, as according to (27) it has the correct scaling in  $m_b$ . Also, the interpolation of the B meson by a local current entails certain differences between the two approaches: the B meson DA  $\phi_B^+$  does not enter LCSR $s$  explicitly and in order to verify the structure of the factorisable term it was necessary to invoke a second sum rule for  $1/\lambda_B$ . It is very likely that any finer details relating to the interactions of the spectator quark in the B, for instance the description of how the soft spectator turns into a collinear one in the  $\pi$ , i.e. any information about the jet functions  $J$  in (26), cannot be resolved in the LCSR approach. Rather than considering that a weakness, I would like to stress that the intention of LCSR $s$  is not so much to study the intricacies of QCD in the heavy quark & large energy limit, and in that respect they certainly cannot compete with SCET, but to provide phenomenologically relevant numerical predictions within a well-defined and controlled framework and in full QCD.

## 4 Numerics

After the more formal discussion in the last section which has demonstrated that LCSR $s$  in the heavy quark limit fulfill all SCET relations and constraints, we proceed to the analysis of numerical aspects.

The numerical value of  $f_+(0)$  is comparatively well known, from both sum rules and lattice calculations; both methods point consistently at the same result,  $f_+^{\text{QCD}}(0) \approx 0.3$  [17, 25]. Let us compare this number with the contribution of the factorisable term in Eq. (26):

$$f_+^{\text{fac}}(0) = \frac{4\pi^2}{3} a_s \frac{1}{\lambda_B} \left\langle \frac{1}{u} \right\rangle_\pi.$$

The first inverse moment of the twist-2  $\pi$  DA can be extracted from experiment [26], at the scale  $\mu \approx 1 \text{ GeV}$ :

$$\left\langle \frac{1}{u} \right\rangle_\pi = 3.3 \pm 0.3.$$

As for the inverse moment of the B meson DA,  $1/\lambda_B$ , it has recently been determined from QCD sum rules as  $\lambda_B = 0.6 \text{ GeV}$  [24]. As this determination corresponds to a tree-level sum rule, the numerical value has to be taken cum grano salis, but should be sufficient for a rough estimate of the factorisable term.  $\alpha_s$  is to be evaluated at the intermediate

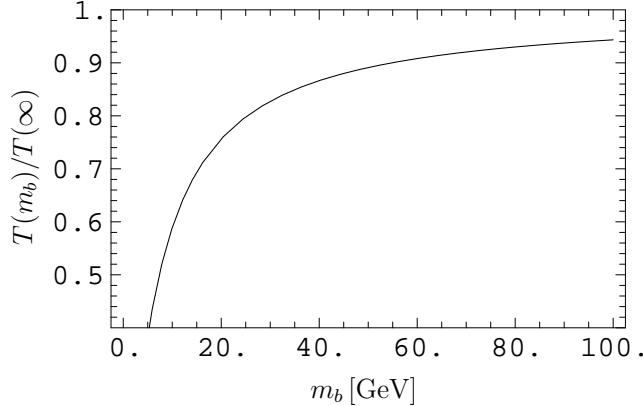


Figure 4: Tree-level twist-2 contribution to  $f_+(0)$  as a function of  $m_b$ . Input parameters:  $\bar{\Lambda} = 0.5$  GeV,  $a_2 = 0$ ,  $\omega_0 = 1$  GeV,  $\tau = 0.7$  GeV.

scale  $\mu^2 \sim \Lambda_{\text{QCD}} m_b$ . To enhance the term as much as possible, I choose  $\mu = 1$  GeV, i.e.  $\alpha_s = 0.51$ . Taking all together, this yields

$$f_+^{\text{fac}}(0) \approx 0.023, \quad (41)$$

i.e. about 10% of the total form factor  $f_+^{\text{QCD}}(0) \approx 0.3$ .

To compare the size of the factorisable term with that of power-suppressed corrections, the obvious thing to do is to derive a sum rule for  $\xi$  (or  $\zeta$ ) and compare with the full QCD result. To NLO in the conformal expansion of DAs I find:

$$\begin{aligned} & f_B m_b^2 \xi(E_{\max}) e^{-\bar{\Lambda}/\tau} = \\ & = 12 f_\pi (1 + 6 a_2^\pi(\mu)) \int_0^{\omega_0} d\omega \omega e^{-\omega/\tau} \left\{ 1 + a_s \left( \pi^2 + 3 - 2 \ln^2 \frac{2\omega}{m_b} - 6 \ln \frac{2\omega}{m_b} \right) \right\} \\ & + 72 f_\pi a_2^\pi(\mu) \int_0^{\omega_0} d\omega \omega e^{-\omega/\tau} a_s \left( \frac{5}{2} - \frac{25}{3} \ln \frac{2\omega}{\mu} \right) \\ & + 2 \mu_\pi^2(\mu) (1 + 30 \eta_3(\mu) - 3 \eta_3(\mu) \omega_3(\mu)) \int_0^{\omega_0} d\omega e^{-\omega/\tau} \\ & + 2 \mu_\pi^2(\mu) a_s \int_0^{\omega_0} d\omega e^{-\omega/\tau} \left( \pi^2 - 4 - 2 \ln^2 \frac{2\omega}{m_b} - 8 \ln \frac{2\omega}{m_b} + 6 \ln \frac{2\omega}{\mu} \right). \end{aligned} \quad (42)$$

In the finite-energy limit  $\tau \rightarrow \infty$ , this agrees with (28). One point to notice is that, although the authors of [11, 14] have argued that the resummation of single and double logarithms in  $\ln m_b/\mu_F$  be relevant, this is not obvious from a phenomenological point of view:  $\sim \ln^2 m_b/(2\omega_0) \sim \ln^2(5/2) \approx 1$  is not really large and in fact much smaller than the nonlogarithmic contributions.

Unfortunately, it turns out that the above sum rule yields  $\xi \approx 2$  in complete disagreement with the expected result  $\sim 0.3$ . The reason is that the expansion of the sum rule for  $f_+(0)$  in  $1/m_b$  is extremely bad at  $m_b = 5$  GeV. Fig. 4 illustrates this feature: I plot the dependence of the tree-level twist-2 contribution to  $f_+(0)$  on  $m_b$ ,

$$T(m_b) \equiv f_B m_b^2 f_+^{T2,\text{QCD}}(0)$$

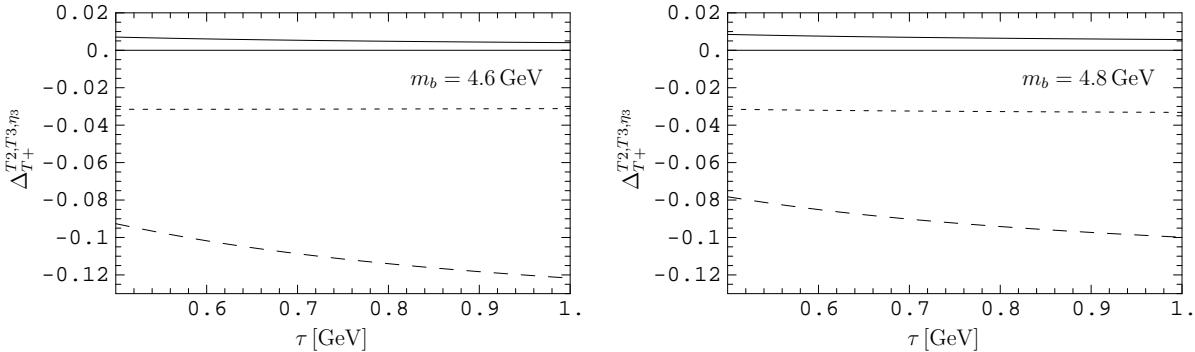


Figure 5: Relative differences  $\Delta_{+T}$  between  $f_T(0)$  and  $f_+(0)$  induced by twist-2 and 3 contributions, respectively, for two different values of the b quark mass. Solid line: radiative corrections to the twist-2 contribution; long dashes: 2-particle twist-2 DAs (asymptotic form), including  $O(\alpha_s)$  corrections; short dashes: contribution of  $\eta_3$ , from both 2 and 3-particle DAs (tree-level).

$$\begin{aligned}
&= -6f_\pi \int_0^{\omega_0} d\omega \exp\left(\frac{\bar{\Lambda} - \omega}{\tau} \left[1 + \frac{\bar{\Lambda} + \omega}{2m_b}\right]\right) \frac{m_b^4}{(m_b + \omega)^3} \left(\frac{m_b^2}{(m_b + \omega)^2} - 1\right) \\
&\times \left(1 + 6a_2 \left(1 - \frac{5m_b^2\omega(2m_b + \omega)}{(m_b + \omega)^4}\right)\right) \tag{43}
\end{aligned}$$

relative to its limiting value for  $m_b \rightarrow \infty$ , as given in the first term on the r.h.s. of Eq. (42). It is obvious that Eq. (42) overshoots the value at the physical quark mass by more than a factor 2. As the calculation of  $\xi$  has, to the best of my knowledge, never been attempted by any other method, it is not clear if Eq. (42) indeed indicates that power-corrections to  $\xi$  are huge, with the asymptotic value only reached at quark masses  $m_b \sim 100$  GeV. A calculation of  $\xi$  in the pure heavy quark limit from lattice, if possible, would surely help to clarify the situation.

As an alternative, one can calculate the difference  $f_T(0) - f_+(0)$  directly from LCSR, which feature an *exact* (i.e. all-order in  $1/m_b$ ) cancellation of the tree-level twist-2 contribution, so the difference should be sensitive to symmetry-breaking effects induced at higher-twist. Let me define (choosing  $\mu = m_b$  for the ultraviolet scale in  $f_T$ )

$$\Delta_{T+}^{(i)} = \frac{f_T^{(i)}(0) - f_+^{(i)}(0)}{f_+^{\text{QCD}}(0)} \tag{44}$$

with  $i \in \{T2, T3, \eta_3, T4\}$ , i.e. the relative contributions to the form factor difference from twist-2, 2-particle twist-3, 3-particle twist-3 and twist-4 contributions, respectively. All these terms are calculated in full QCD, with no  $1/m_b$  expansion; for  $f_T$ , the radiative corrections to the twist-3 contributions are new. The input in the corresponding sum rules is the same as in Ref. [17]: the b quark mass (one-loop pole mass) is fixed at 4.6 (4.8) GeV. The continuum threshold  $\omega_0 = 1.25$  GeV, i.e.  $s_0 = 34$  GeV<sup>2</sup>, together with the Borel-parameter window  $0.5 \text{ GeV} < \tau < 1 \text{ GeV}$  yields  $f_B = 0.195$  GeV, in good agreement with lattice results. I fix  $\mu = \sqrt{m_B^2 - m_b^2}$ , i.e. it is formally of order  $\sqrt{m_b}$ .

The results are shown in Fig. 5.  $\Delta_{T+}^{T2}$  is formally of leading order in  $1/m_b$ , but  $O(\alpha_s)$ , and actually is very small numerically,  $\sim 1\%$ . This term includes in particular the hard symmetry-breaking terms in (24), which numerically cancel to a large extent. The 2-particle twist-3 term is formally  $O(1/m_b)$ , but starts at tree-level. It is numerically the most dominant effect, a nonfactorisable symmetry-breaking term that cannot be calculated in SCET (cf. also Ref. [12]). The (tree-level) contribution of the quark-quark gluon matrix element  $\eta_3$  that enters both nonleading terms in the conformal expansion of  $\phi_p$  and  $\phi_\sigma$  and the leading term in  $\mathcal{T}$  is shown separately, as  $\Delta_{T+}^{\eta_3}$ ; it is small and corrects the form factor by about 3%. Finally, and not shown in the plot, the contribution of twist-4 DAs is of about the same size as the factorisable twist-2 ones: 1%. The most important contribution still missing are radiative corrections to the 3-particle twist-3 DA, which are formally of leading order in  $1/m_b$  for  $f_T$  and  $f_+$  separately, but expected to be spin-symmetric, i.e. of order  $\alpha_s/m_b$  in the difference of form factors.

A similar analysis could be performed for  $\Delta_{-+}$ , but even without that the overall emerging picture is quite clear: nonfactorisable symmetry-breaking corrections are the numerically most relevant ones, whereas factorisable contributions are small.

The above results should not be interpreted as a new determination of  $f_T(0)$ ; updates for all form factors will be published separately.

## 5 Summary and Conclusions

In this paper I have discussed, in quite some detail, the relation between SCET factorisation formulas for the  $B \rightarrow \pi$  form factors  $f_{+,0,T}$  and their light-cone sum rules. I have demonstrated that to  $O(\alpha_s)$  accuracy and for contributions from 2-particle twist-2 and 3 distribution amplitudes, light-cone sum rules in the large energy limit  $m_b \rightarrow \infty$ ,  $E_\pi \rightarrow \infty$  fulfill all symmetry relations predicted by SCET. I have also demonstrated that the non-factorisable contributions from 2-particle twist-3 DAs are spin-symmetric to leading order in  $1/m_b$ , but break spin-symmetry at higher order. Numerically they induce a splitting between  $f_+(0)$  and  $f_T(0)$  of about 10% and are the most relevant symmetry-breaking corrections.<sup>3</sup> Within the SCET approach, these contributions correspond to subleading soft form factors and cannot be resolved any further, cf. Ref. [12]. For the chosen form factor difference  $f_T - f_0$ , the hard gluonic corrections cancel to a large extent; this need not be the case for other form factor differences, where these corrections may even compete with the subleading ones. The results of my study suggest, however, that subleading soft form factors are, in general, likely to be numerically relevant in form factor relations, also for  $B \rightarrow$  vector meson decays.

The motivation for this study was the question if SCET predictions can help to get a better grip on  $B$  decays. There is no doubt that SCET constitutes a major step forward and has transformed the way factorisation proofs will be done, with its shift from a diagrammatical level to all-order proofs based on field transformations in the Lagrangian. However, in the context of for instance electromagnetic hadron form factors, experience has shown that factorisation is often broken at subleading level, i.e. that the clean separation between hard and soft contributions is spoiled at the first nontrivial order in the

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<sup>3</sup>This is not a statement about the actual splitting between these two form factors, but about its contribution from 2-particle twist-3 DAs. A more comprehensive study is underway.

expansion. Usually this is not a problem from a phenomenological point of view as, if the momentum transfer is large enough, experiments are insensitive to these power-suppressed effects. The situation is different in B decays: the expansion parameter,  $1/m_b$ , is not a dynamical variable, but fixed and not very small. Experience with HQET, the heavy quark effective theory, has shown that  $1/m_b$  corrections in general cannot be neglected, and one of the most important applications of HQET in B physics, the extraction of  $|V_{cb}|$  from  $B \rightarrow D^* e \nu$  at zero recoil, relies precisely on the fact that linear corrections in  $1/m_b$  are absent, by virtue of Luke's theorem. In the application of SCET to heavy-to-light decays, nonfactorisable terms are present already at leading order<sup>4</sup> and make it impossible to calculate for instance  $f_+$  directly. Still, one may expect to express the ratio of form factors in terms of perturbatively calculable coefficients. The results I have presented in this paper indicate that this expectation is not realised. The same is likely to be the case also for other B decay processes treated in a similar way, in particular nonleptonic decays.

Let me conclude with a comment on the prospects of relating  $B \rightarrow \pi$  to  $B \rightarrow K$  form factors via SU(3) symmetry. As  $\zeta(E_{\max})$  is essentially proportional to  $-f_\pi \phi'_\pi(1)$ , one expects an SU(3) breaking ratio

$$\frac{\zeta_K(E_{\max})}{\zeta_\pi(E_{\max})} \equiv 1 + \Delta_{\text{SU}(3)}^\zeta(\mu) \approx \frac{f_K \phi'_K(1, \mu)}{f_\pi \phi'_\pi(1, \mu)} = \frac{f_K}{f_\pi} \frac{2 + \sum_{n=1}^{\infty} a_n^K(\mu)(n+1)(n+2)}{2 + \sum_{n=1}^{\infty} a_n^\pi(\mu)(n+1)(n+2)}, \quad (45)$$

where we have expressed  $\phi(u)$  by its conformal expansion

$$\phi(u, \mu) = 6u(1-u)(1 + \sum_{n=1}^{\infty} a_n(\mu) C_n^{3/2}(2u-1)).$$

The first two Gegenbauer moments  $a_1^K$  and  $a_2^K$  have recently been redetermined in Ref. [27]:

$$a_1^K(1 \text{ GeV}) = -0.18 \pm 0.09, \quad a_2^K(1 \text{ GeV}) = 0.16 \pm 0.10.$$

For the  $\pi$ , there are experimental data available as well as sum rule calculations. Taking the number quoted in a recent paper, Ref. [28],  $a_2^\pi(1 \text{ GeV}) = 0.19$  ( $a_1^\pi = 0$  up to isospin-breaking effects), one has

$$\Delta_{\text{SU}(3)}^\zeta(1 \text{ GeV}) = -0.18,$$

which indicates nonnegligible SU(3) breaking. Although this number comes with a large uncertainty, as it is very sensitive to higher order Gegenbauer moments, it indicates that the actual SU(3) breaking can be large and potentially even have different sign than the naive expectation  $\Delta_{\text{SU}(3)}^\zeta \approx f_K/f_\pi - 1 = 0.2$ .

## Acknowledgements

I would like to thank the participants of the Ringberg Workshop on Heavy Flavours, Ringberg April 2003, and FPCP, Paris June 2003, for stimulating presentations and discussions, which contributed significantly to the genesis of the considerations presented in this paper.

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<sup>4</sup>The reason being that, technically speaking, the operators entering the factorisable part in (26) are subleading in the expansion parameter of the SCET Lagrangian, so that it would be more precise to say: factorisable contributions are *absent* at leading order.

## A Light-Cone Sum Rules in the Heavy Quark Limit

In this appendix I give the explicit LCSR for  $f_-(0)$  and  $f_T(0)$  in the combined heavy quark and finite energy limit, for 2-particle twist-2 and 3 contributions to  $O(\alpha_s)$  accuracy. Use of the equation of motion relation  $6\phi_p(1) = -\phi'_\sigma(1)$  is implied. I use the notations  $\mu_\pi^2 = f_\pi m_\pi^2 / (m_d + m_u) = -2\langle\bar{q}q\rangle/f_\pi$  and  $a_s = C_F \alpha_s / (4\pi)$ .

$$f_B m_b f_-^{T2}(0) = f_\pi \frac{\omega_0^2}{m_b} \phi'_\pi(1) \left\{ 1 + a_s \left( \pi^2 - 2 \ln^2 \frac{2\omega_0}{m_b} - 4 \ln \frac{2\omega_0}{m_b} + 2 \ln \frac{2\omega_0}{\mu} \right) \right\} \\ - 4a_s f_\pi \frac{\omega_0^2}{m_b} \left( \ln \frac{2\omega_0}{\mu} - 1 \right) \left\{ \int_0^1 du \frac{\phi_\pi(u) + \bar{u}\phi'_\pi(1)}{\bar{u}^2} + \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} \right\}, \quad (\text{A.1})$$

$$f_B m_b f_-^{T3}(0) = -2\mu_\pi^2 \frac{\omega_0}{m_b} \phi_p(1) \left\{ 1 + a_s \left( \pi^2 - 8 - 2 \ln^2 \frac{2\omega_0}{m_b} - 4 \ln \frac{2\omega_0}{m_b} + 6 \ln \frac{2\omega_0}{\mu} \right) \right\} \\ - 2\mu_\pi^2 \frac{\omega_0}{m_b} a_s \left( 4 \ln \frac{2\omega_0}{\mu} - 3 \right) \int_0^1 du \frac{1}{\bar{u}} (\phi_p(u) - \phi_p(1)) \\ + \mu_\pi^2 \frac{\omega_0}{3m_b} a_s \int_0^1 du \frac{1}{\bar{u}^2} (\phi_\sigma(u) + \bar{u}\phi'_\sigma(1)), \quad (\text{A.2})$$

$$f_B m_b f_T^{T2}(0) = -f_\pi \frac{\omega_0^2}{m_b} \phi'_\pi(1) \left\{ 1 + a_s \left( 3 + \pi^2 - 2 \ln^2 \frac{2\omega_0}{m_b} - 6 \ln \frac{2\omega_0}{m_b} + 4 \ln \frac{2\omega_0}{\mu} \right) \right\} \\ + 4a_s f_\pi \frac{\omega_0^2}{m_b} \left( \ln \frac{2\omega_0}{\mu} - 1 \right) \left\{ \int_0^1 du \frac{\phi_\pi(u) + \bar{u}\phi'_\pi(1)}{\bar{u}^2} + \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}} \right\}, \quad (\text{A.3})$$

$$f_B m_b f_T^{T3}(0) = 2\mu_\pi^2 \frac{\omega_0}{m_b} \phi_p(1) \left\{ 1 + a_s \left( \pi^2 - 5 - 2 \ln^2 \frac{2\omega_0}{m_b} - 6 \ln \frac{2\omega_0}{m_b} + 8 \ln \frac{2\omega_0}{\mu} \right) \right\} \\ + 2\mu_\pi^2 \frac{\omega_0}{m_b} a_s \left( 4 \ln \frac{2\omega_0}{\mu} - 3 \right) \int_0^1 du \frac{1}{\bar{u}} (\phi_p(u) - \phi_p(1)) \\ - \mu_\pi^2 \frac{\omega_0}{3m_b} a_s \int_0^1 du \frac{1}{\bar{u}^2} (\phi_\sigma(u) + \bar{u}\phi'_\sigma(1)). \quad (\text{A.4})$$

## B Hard spectator contribution to $f_+$

We have

$$\bar{B}(q^2 = 0) = \frac{i}{(4\pi)^2} \frac{1}{\bar{u}p_B^2} \left\{ \left( \frac{m_b^2}{up_B^2} - 1 \right) \ln \left( 1 - u \frac{p_B^2}{m_b^2} \right) - \left( \frac{m_b^2}{p_B^2} - 1 \right) \ln \left( 1 - \frac{p_B^2}{m_b^2} \right) \right\}.$$

In the finite-energy limit, the continuum-subtracted Borel transform is

$$\hat{B}_{\text{sub}}^\infty \bar{B} = \frac{i}{(4\pi)^2} \left[ \frac{1}{\bar{u}} (\ln u_0 + \bar{u}_0) \Theta(u_0 - u) + \frac{1}{u\bar{u}} (u \ln u + \bar{u}u_0) \Theta(u - u_0) \right]$$

with  $u_0 = m_b^2/s_0$ . Upon convolution with  $\phi_\pi(u)$  and using the scaling law (27), the second term yields  $8/3 \omega_0^3/m_b^3 \phi'_\pi(1)$ , i.e. a power-suppressed term, whereas the first one yields

$$\bar{B} \rightarrow -\frac{i}{(4\pi)^2} \frac{2\omega_0^2}{m_b^2} \int_0^1 du \frac{\phi_\pi(u)}{\bar{u}}.$$

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